UIMP, 13 agosto, 2007

- COMPLEJIDAD Y CAOS EN LOS MODELOS ECONÓMICOS

- Pablo Coto Millán

Breve Historia del EGC

- (1776-1900) Primera Etapa. De la Mano Invisible a la Noción Walrasiana
- (1900-1954) Segunda Etapa. De la Noción Walrasiana a la Prueba de Existencia del EGC de Arrow-Debreu
- (1954-1974) Tercera Etapa. De la Prueba de Existencia a las de Unicidad y Estabilidad de Arrow-Debreu

Propiedades Positivas del Modelo de EGC

- i) Físicamente factibles
- ii) Técnica y asignativamente eficientes. (1er.Tª Bienestar el equilibrio es OP)
- iii) Insesgadas. Cualquier estado óptimo puede ser alcanzado desde un modelo de EGC. (2do. Tª Bienestar)

Las dotaciones de recursos son un dato. La distribución resultante de modelo de EGC es ajena a Equidad o Justicia Distributiva

Breve Historia de los Fallos de Mercado

*Problemas de Asignación de Derechos de Propiedad: Bienes Públicos, Recursos Naturales, Recursos de Propiedad Común y Externalidades

Problemas de Negociación y Elección entre agentes: Costes Decrecientes (Monopolios naturales), Negociación y elección entre patronal y sindicatos

Problemas de Información: Principal y Agente, Riesgo Moral, Selección Adversa y Señales.

Breve Historia sobre los Fallos del Gobierno

- Problemas de Asignación de Derechos de Propiedad del Gobierno: Bienes Públicos, Bienes Privados y Corrupción.
- Problemas de Negociación y Elección: Imposibilidad de Agregar preferencias=negociación, Paradoja del Orden de las Votaciones, Paradoja del Votante Medio, Paradoja del Partido Bisagra, etc
- Problemas de Información: Principal-Agente, Concursos, Subastas, Señales, etc.

2. Other Models of General Equilibrium

2.1 Model 1

This model is based on Baumol's version (1970). This version was studied and partly modified by Day (1983) with the aim of showing how fluctuations of an erratic and unstable nature can appear in the population. The previous version by Day has been slightly altered so that the model enables us to obtain a stationary solution as well as stable periodic results and chaotic results.

Below we offer an introduction to the application of the concept of complexity in an economy. However, two things may be surprising. First, the nature of the complex mathematics itself and secondly, the capacity of a non-linear model to generate unusually different results without any variation in the model.

2.1 Model 1

2.1.1 Assumptions

i) We assume an economy with only one sector (agriculture) where the total product Y_t depends on the amount of working population P_t.

$$Y_t = f(P_t), \text{ con } f'(P_t) > 0 \text{ y } f''(P_t) < 0$$

where $f'(P_t) > 0$ expresses the positive character of the marginal product of labor and where $f''(P_t) < 0$ indicates the existence of decreasing returns. We assume that $f(0) = 0$ and that the function is continuous and concave.

ii) We assume that the total product Y_t, is distributed according to the mean product. In other words, we assume that the wage is equal to the mean productivity.

$$w_t = f(P_t)/P_t$$

2.1 Model 1

2.1.1 Assumptions

iii) We assume that the population growth rate in per capita terms is ruled by function:

$$\Delta P_t/P_t = \min[\lambda, (w-\sigma)/\sigma]$$

where $\Delta P/P$ is the population growth rate, λ is the maximum growth rate of population which enables the most advantageous biological conditions and $(w-\sigma)/\sigma$ is the maximum growth rate which permits the current availability of food, such availability culturally determined by the current wages. When wages are at a subsistence level $w = \sigma$ the rate is null and population remains stationary. The level of subsistence σ can be considered as determined by sociological or cultural factors.

2.1 Model 1

2.1.2 Development of the Model

Under the previous assumptions, the function which rules the population growth is:

$$\Delta P/P = \min [\lambda, (w-\sigma)/\sigma] \qquad (1.1)$$

Given this, if we substitute the statement in (1.1) we obtain:

$$(P_{t+1} - P_t) / P_t = \min [\lambda, (w-\sigma)/\sigma]$$
$$P_{t+1} / P_t = \min [(\lambda + 1); w/\sigma]$$
$$P_{t+1} = \min [(\lambda + 1) P_t; (w/\sigma) P_t] \qquad (1.2)$$

this equation describes the evolution of a sequence of generations.

Under ii) assumption we can substitute in (1.2) and obtain:

$$P_{t+1} = \min [(\lambda + 1) P_t; f (P_t)/\sigma)] \qquad (1.3)$$

2.1 Model 1

2.1.2 Development of the Model

This equation shows that the population growth is determined by two regimes. Under the first regime the population growth rate is controlled by biological rate λ while under the second regime it is controlled by the maximum rate with level of subsistence σ. These two population growth rates give rise to what we call two phases of population: biological and subsistence.

2.1.3 The Solution

Two possible results of the model are:

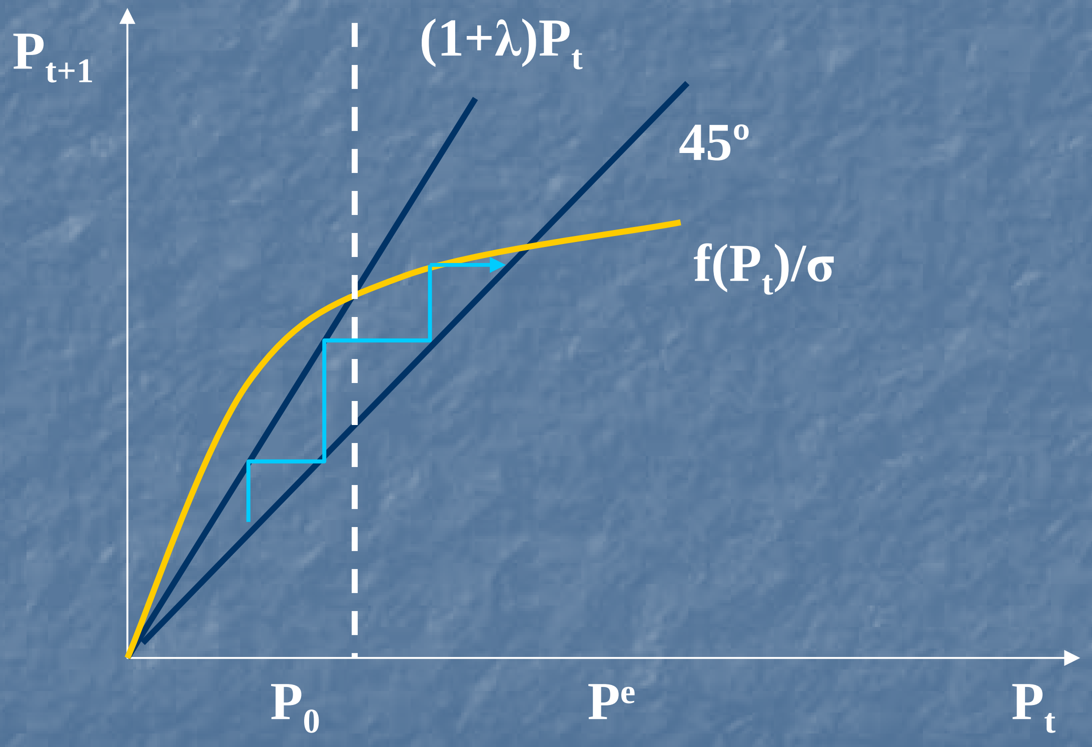
a) The stationary solution.

b) Stable cycles of different periods.

c) Chaotic results.

2.1 Model 1

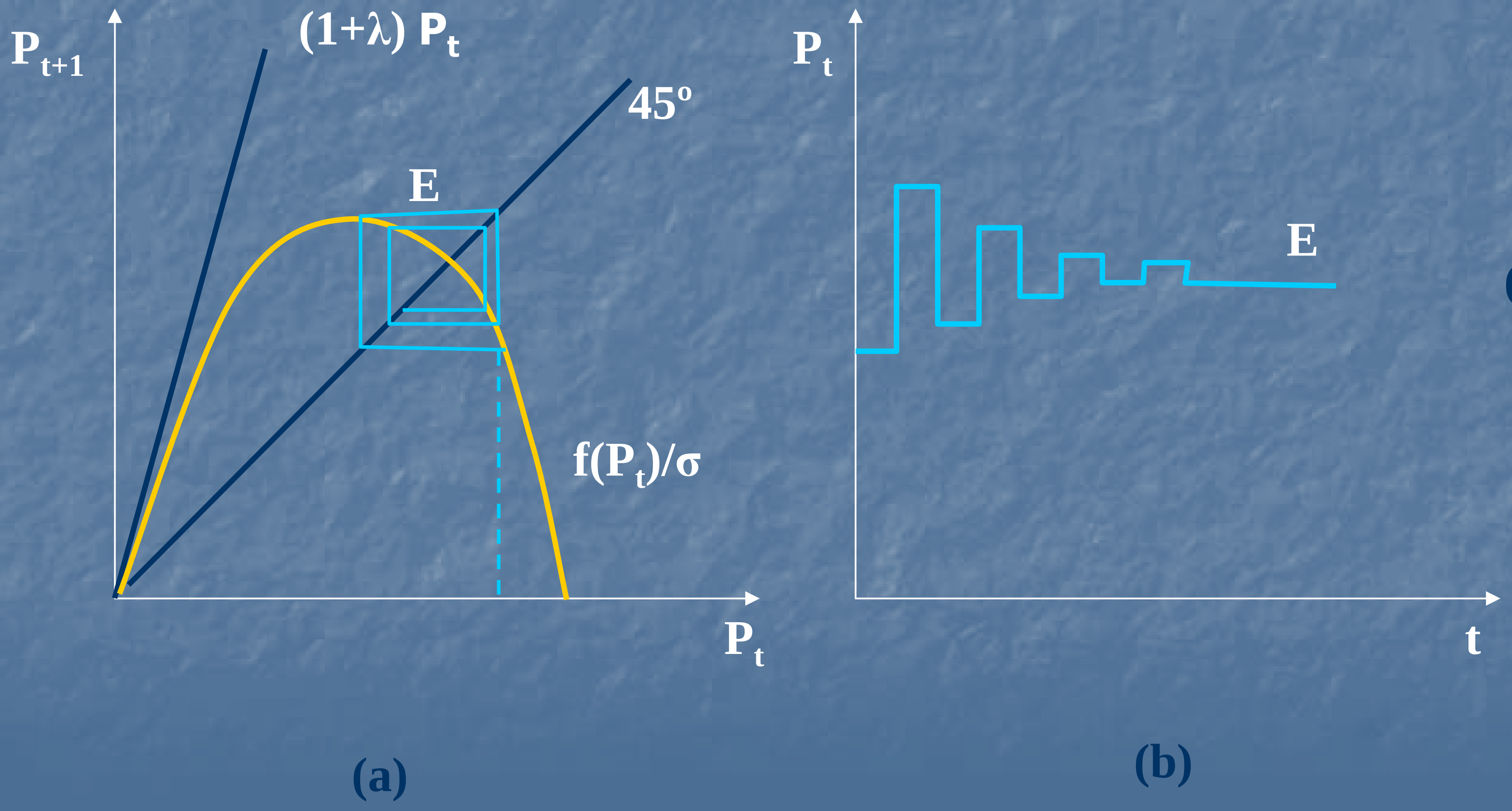
2.1.3 The Solution



(2.1)

2.1 Model 1

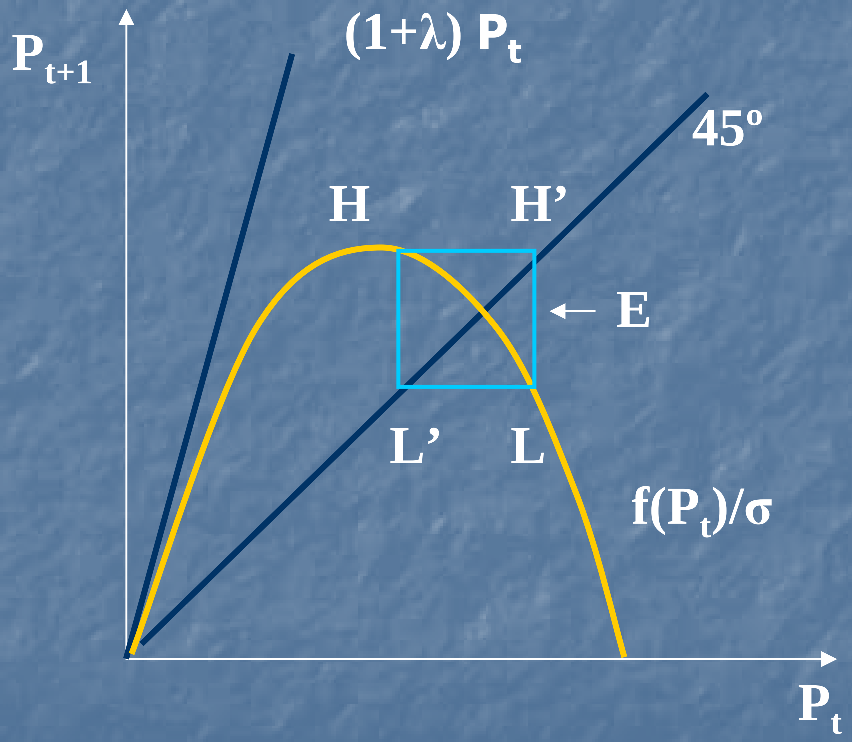
2.1.3 The Solution



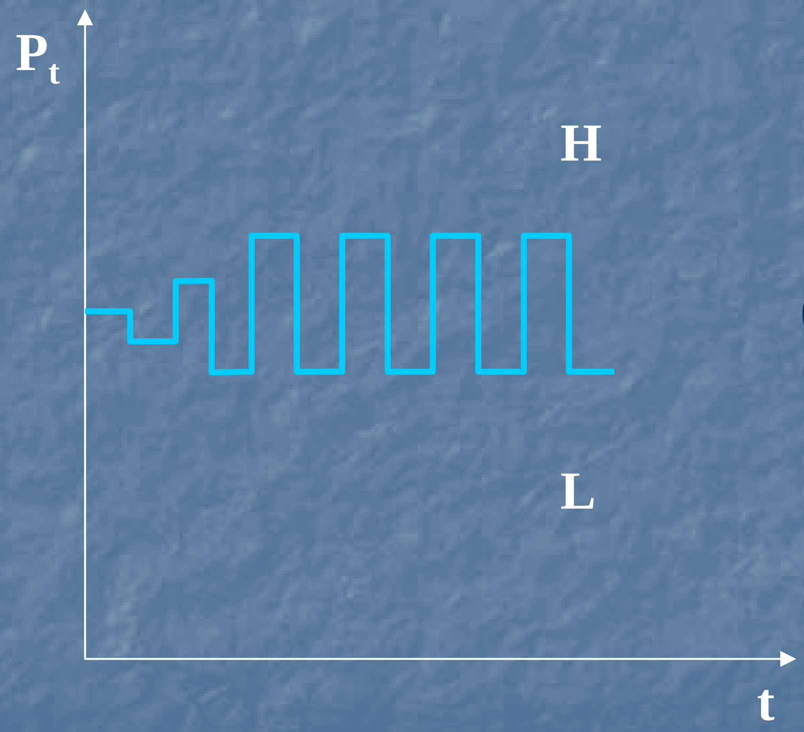
(2.2)

2.1 Model 1

2.1.3 The Solution



(a)

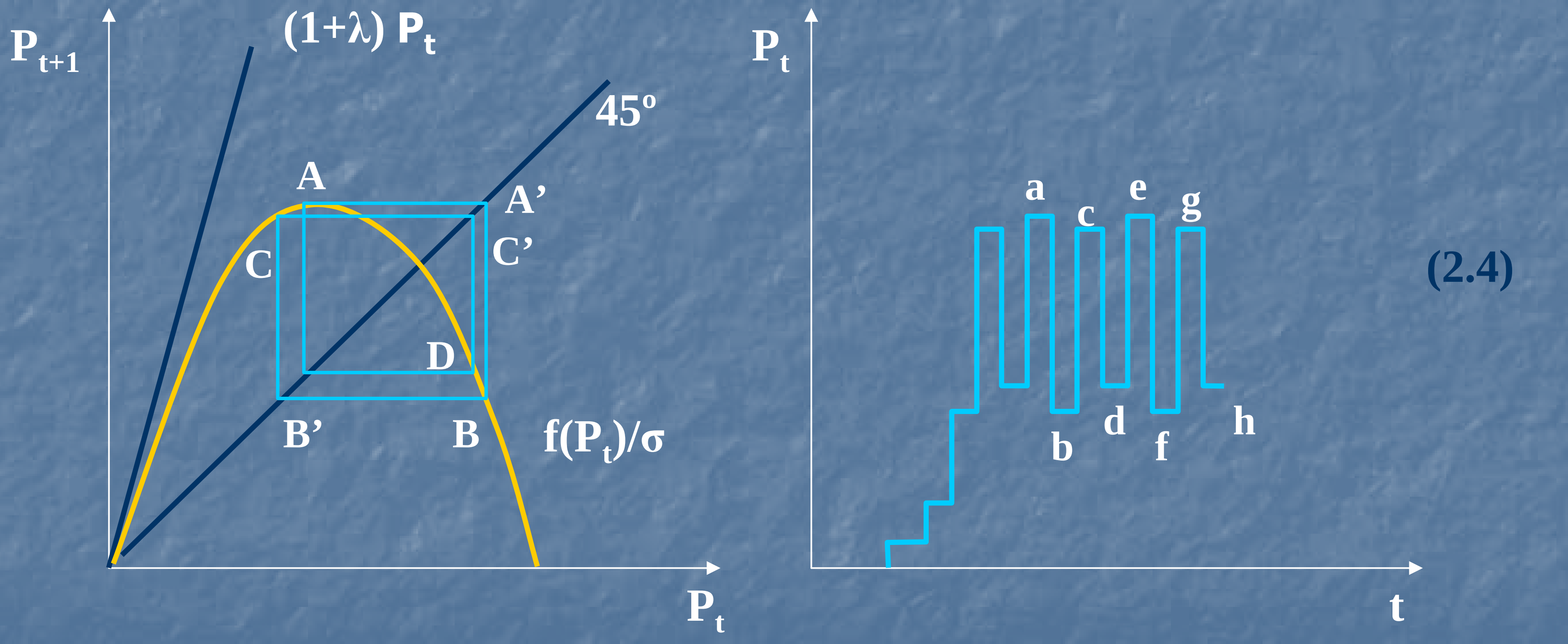


(b)

(2.3)

2.1 Model 1

2.1.3 The Solution

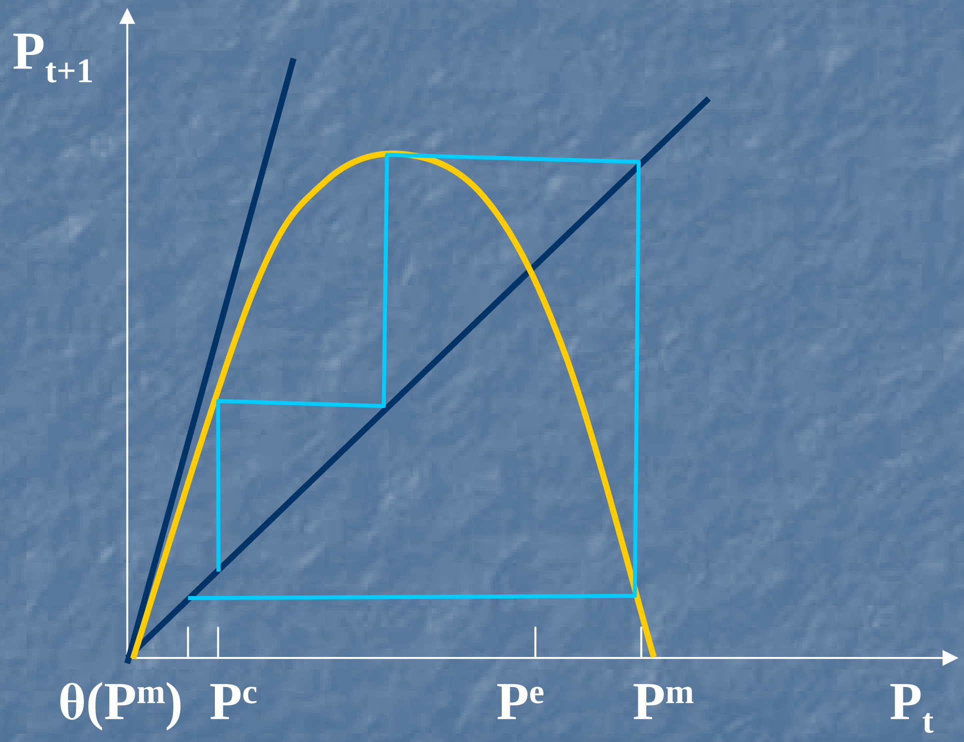


(a) (b)

(2.4)

2.1 Model 1

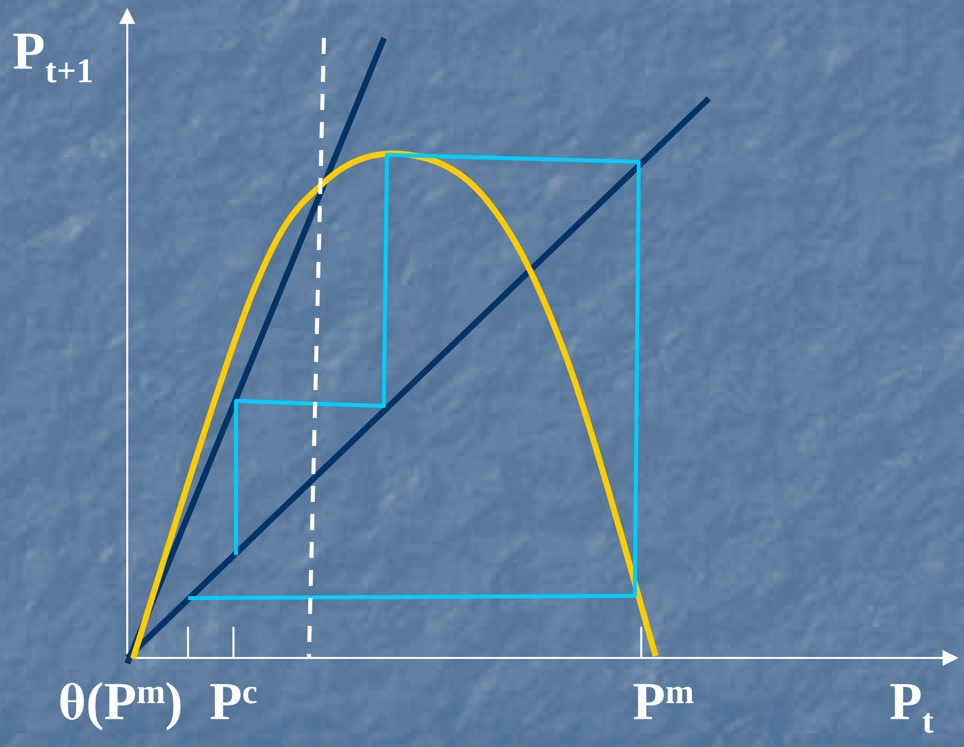
2.1.3 The Solution



(2.5)

2.1 Model 1

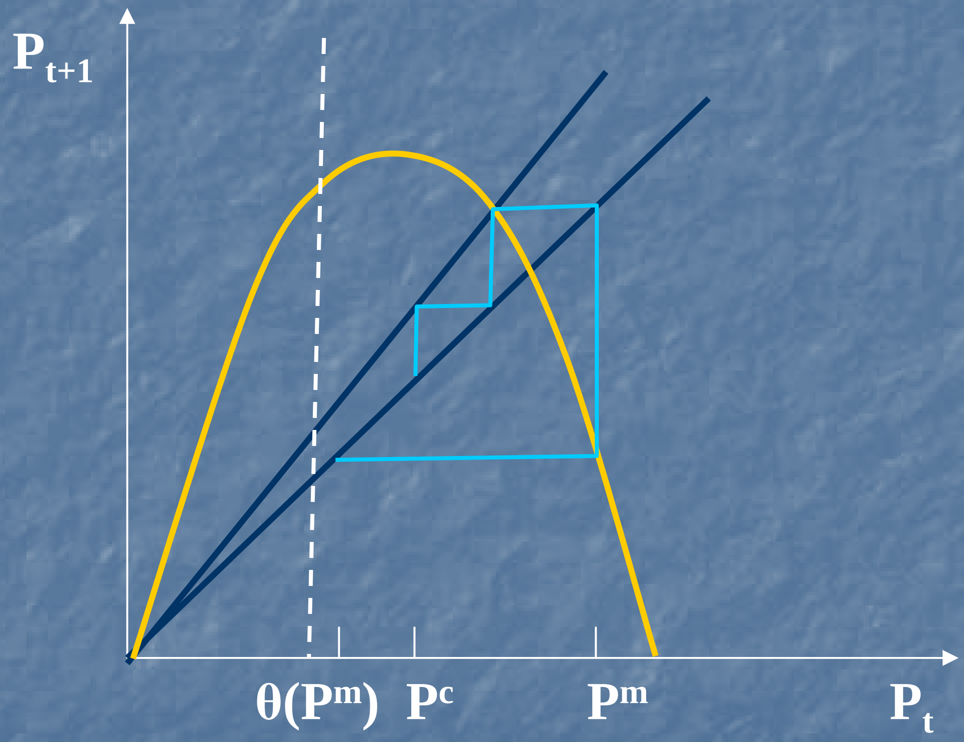
2.1.3 The Solution



(2.6)

2.1 Model 1

2.1.3 The Solution



(2.7)

How can we know that there is chaos in these cases? We know this through the Li-Yorke theorem enunciated below.

2.1 Model 1

2.1.4 The Li-Yorke Theorem (Baumol and Benhabid's Version, 1989)

Assume that we have a continuous function in differences of the type $P_{t+1}=f(P_t)$ and that there is some interval (a,b) such that the whole cycle initiated within said interval never exceeds the limits. If this assumption, - which does not seem restrictive and in which $P_3 < P_o < P_1 < P_2$ - is satisfied, we can state the following:

1. For any whole number k > 1 we can find at least one initial condition (in other words, a value P_o) which generates cycles of period k. This implies that there are virtually infinite cyclic-possibilities which do not lead to equilibrium.

2. Within interval (a,b) there is a disordered and uncountable set of points such that any pair of initial curves within said set may become infinitely close together at any point to later separate from each other but they never converge between themselves or with any other type of regular or periodic curve.

In the growth model we are analyzing the above theorem is satisfied as can be appreciated in figures 2.5, 2.6 and 2.7.

2.2 Model 2

The model here presented is based on the work by Diamond (1965) and is used together with Benhabid and Day's further version (1980 and 1982).

2.2.1 Assumptions of the Model

i) Assume that individuals live two time periods t and t+1 at a time t and two generations exist at the same time: 0 and 1. Only young people work, who save part of their income and are actually the ones who acquire capital stock with their savings. The capital stock available for the individual who invests during the period t+1 is called k_{t+1} and equals the capital stocks saved by the subject during period t.

$$k_{t+1} = w_t - c_0(t)$$

In the above statement, w_t is the wage and c_o (t) is the individual's consumption born in generation 0 during period t. We also assume that the capital stock of each period is wholly consumed within that period. This implies that the capital stock available during t+1 equals that invested in period t.

$$I_t = k_{t+1}$$

2.2 Model 2

2.2.1 Assumptions of the Model

ii) Assuming that an individual who represents the younger generation only receives income from work, this individual earns a wage w which he must distribute in the two periods of his life t and t+1. This means that his consumption during period t+1 will be given by the following statement:

$$c_0(t+1) = [w_t - c_0(t)]\cdot(1 + r_{t+1})$$

where r_{t+1} is the interest rate of savings from period t to t+1. In other words, the economic agent can save and, through the credit market, transfer resources from the first period of life to the second.

iii) Assume that the population of both generations remains always the same.

2.2 Model 2

2.2.1 Assumptions of the Model

iv) Assume that the utility function for each generation is of the Cobb-Douglas type and is given by:

$$u_t = c_0(t)^{\beta} \cdot c_0(t+1)^{(1-\beta)}$$

or in logarithmic form:

$$\text{Ln } u_t = \beta \text{ Ln } c_0(t) + (1 - \beta) \text{ Ln } c_0(t+1)$$

Here β is a parameter which measures the subject's availability in order to replace present consumption with future consumption. The higher β is the higher the marginal rate of substitution between present and future consumption will be (in other words, the $dc_o(t+1)/dc_o(t)$ ratio in absolute value). It is assumed that β increases as wage increases. That is to say:

$$\beta = \beta(w) = a \cdot w_t$$

being $a > 0$.

v) Assume that individual production function is of the Cobb-Douglas type:

$$f(k_t) = A \cdot k_t^{\alpha}.$$

2.2 Model 2

2.2.2 Development of the Model

2.2.2.1 Individuals' Behavior: Inter-temporal Maximization

The individuals who represents the initial generation 0 in period t will maximize their utility function given a level of wages and an market interest rate and they will assign their consumption in such a way that:

$$\frac{\partial u_t / \partial c_0(t)}{\partial u_t / \partial c_0(t+1)} = (1 + r_{t+1})$$

$$c_0(t+1) = [w_t - c_0(t)] \cdot (1 + r_{t+1})$$
$$c_0(t) = \theta\,(w_t, r_{t+1})$$
$$c_0(t+1) = \psi\,(w_t, r_{t+1})$$

From above we derive that-:

$$w_t - \theta\,(w_t, r_{t+1}) = s_t$$
$$s_t = s\,(w_t, r_{t+1})$$

2.2 Model 2

2.2.2 Development of the Model

2.2.2.1 Individuals' Behavior: Inter-temporal Maximization

Therefore, the saved amount may be expressed as a function of the relevant levels of wage and interest:

$s_t = s(w_t, r_{t+1}); 0 < s_w < 1; s_r$ **indeterminate**

Companies hire work and capital in perfect competition, therefore, it will be verified that the respective marginal productivity will fix the prices of capital and wages.

$$f'(k_t) = r_t$$
$$f'(k_t) - k_t \cdot f'(k_t) = w_t$$

where k_t is the capital-labor ratio of companies.

2.2 Model 2

2.2.2 Development of the Model

2.2.2.1 Individuals' Behavior: Inter-temporal Maximization

Equilibrium in goods market requires that the demand for goods equals supply within the same period. This implies that saving must equal investment in aggregate terms and the fact that all the accumulation of k comes from youngsters means that savings per young individual must equal investment per young individual. That is to say:

$$k_{t+1} = s\,[(w_t, (1 + r_{t+1})]$$

Equilibrium in the input market also requires that the wage rate and the capital interest ratio of firms equal the respective marginal products.

2.2 Model 2

2.2.2 Development of the Model

2.2.2.2 Equilibrium Dynamics and State

Utility maximization subjected to budget restriction implies that:

$$\frac{\partial u_t / \partial c_0(t)}{\partial u_t / \partial c_0(t+1)} = (1 + r_{t+1})$$

$$\partial u_t / \partial c_0(t) = \beta / c_0(t)$$
$$\partial u_t / \partial c_0(t+1) = (1 - \beta) / c_0(t+1)$$

Then:

$$\frac{\beta / c_0(t)}{(1-\beta) / c_0(t+1)} = (1 + r_{t+1})$$

By assumption ii):

$$c_0(t+1) = [w_t - c_0(t)] \cdot (1 + r_{t+1})$$

2.2 Model 2

2.2.2 Development of the Model

2.2.2.2 Equilibrium Dynamics and State

By substituting:

$$\frac{\beta / c_0(t)}{(1-\beta)/[w_t - c_0(t)] \cdot (1+r_{t+1})} = (1+r_{t+1})$$

$$\beta / (1 - \beta) = c_0(t) / [w_t - c_0(t)] = (w_t - k_{t+1}) / k_{t+1}$$
$$\beta \cdot k_{t+1} = (1 - \beta)\cdot(w_t - k_{t+1})$$
$$\beta \cdot k_{t+1} = w_t - k_{t+1} - \beta \cdot w_t + \beta \cdot k_{t+1}$$
$$k_{t+1} = w_t \cdot (1 - \beta)$$

Given the assumption that $\beta = a \cdot w_t$ we can write:

$$k_{t+1} = w_t \cdot (1 - a \cdot w_t) \qquad (2.4)$$

By substituting the equilibrium conditions in the input market and the definition of each production function:

$$k_{t+1} = A \cdot k_t^{\alpha} \cdot (1 - \alpha) \cdot [1 - a \cdot A \cdot k_t^{\alpha} \cdot (1 - \alpha)]$$

2.2 Model 2

2.2.3 The Solution

The above equation has a single and stable solution as long as:

$$\left|\frac{dk_{t+1}}{dk_t}\right| < 1$$

How can we know whether this is satisfied?

By estimating:

$$\frac{dk_{t+1}}{dk_t} = \alpha A k_t^{\alpha-1}(1-\alpha) - a2\alpha A^2 k_t^{2\alpha-1}(1-\alpha)^2$$

We ignore whether the absolute value of this derivative is higher, lower than or equal to 1. This will depend on the values of A, a and α. Nothing can be ensured without more restrictions about the course of the accumulated capital stock. However, we can guarantee that chaotic dynamics can appear in the model for certain values of A, a and α since Li-York's sufficient condition is satisfied.

2.2 Model 2

2.2.3 The Solution

For example, for A = 80, a = 1 and α = 0,95, with K_o = 0,0384 we obtain from equation (11.4) the following values:

$$K_1 = 0{,}148105$$
$$K_2 = 0{,}2269627$$
$$K_3 = 0{,}0217787$$

Therefore:

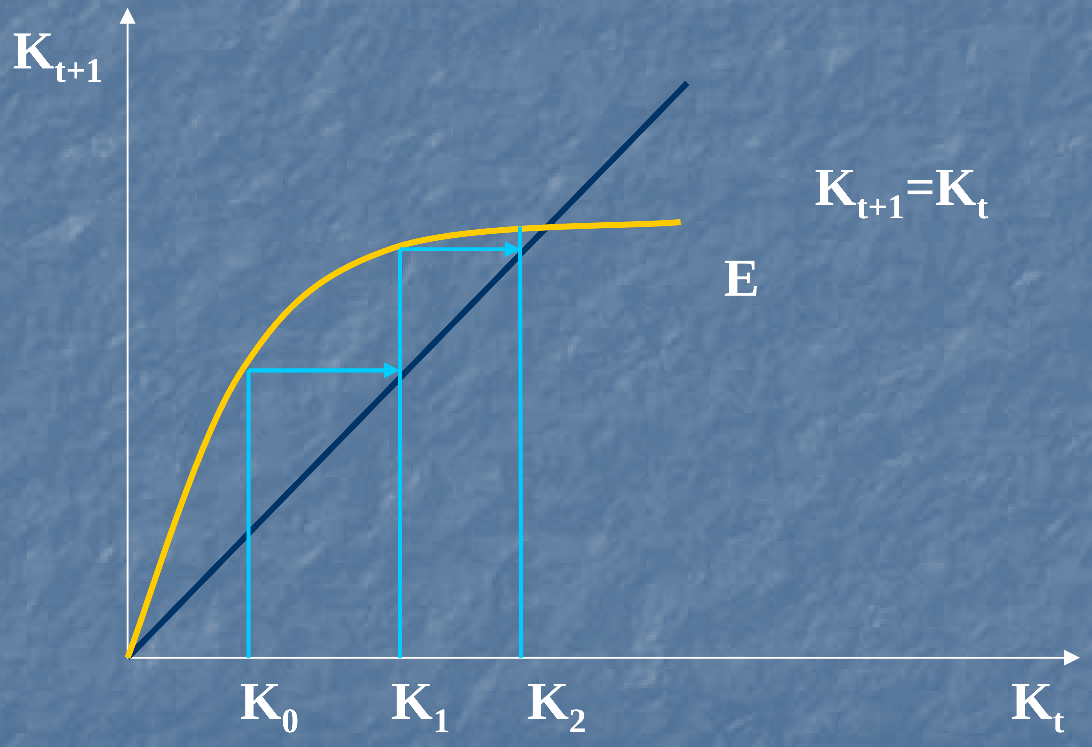
$$K_3 < K_0 < K_1 < K_2$$

Then, Li-York's sufficient condition is verified.

2.2 Model 2

2.2.3 The Solution

Graphically, we will have equilibrium state when:



(2.8)

2.3 Final Remarks

Two conclusions are reached from the above analysis taking into account the restrictions of the initial hypotheses:

First; the significant changes produced in the behavior of an economy do not prevent us from understanding how it works. In spite of the fact that the economy may be as unpredictable as the weather, we can still understand how it operates.

Second; the existence of complex dynamics gives rise to many doubts about predictions in economy.